FLOW OF A GAS IN AN AXISYMMETRIC CAVITY

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Using the basic assumptions of [1], an investigation is made of the problem of flow of a jet in an axisymmetric cavity. The velocity field in the region where the jet is turned is determined.

We shall consider the axisymmetric steady problem of the propagation of a turbulent jet in a closed cavity (Fig. 1). As was done in [1], we shall divide the whole flow region into two parts: the first, K'M'MK, in which the turbulent jet propagates into an opposing stream of fluid, and the second, M'O'OM, in which there occurs a turning about of the streamlines, approximately according to the laws of motion of an ideal fluid. In the first part of the flow in the mixing zone of thickness b, which is in essence a boundary layer formed under the opposing motion of the fluid jets, the flow velocity changes in magnitude and direction from $-U_0$ (in the initial part of the jet) or $-U_m$ (in the main part of the jet) to U_n. According to [1] we shall consider the velocity profile in the mixing zone of the main part of the jet as being given by the formula

$$(U_x - U_n)/(U_m - U_n) = [1-(y/b)^{1.5}]^2 . \tag{1}$$

The mixing zone thickness b appearing in (1) has the form

$$b = c(x_0 - x), \quad x \leqslant x_0, \tag{2}$$

where c is a turbulence constant (as is true also for auxiliary jets, c = 0.22 for the main part of the jet).

For the turning section MM', in the chosen coordinate system, we may write, in conformity with [2]

$$x_0 - x_M = 4.25R; \quad U_{nM} = 0.3U_0;$$

$$U_{mM} = -\frac{2.92 b_0}{\sqrt{R^2 - b_0^2}} U_0. \tag{3}$$

In the second part of the axisymmetric flow, turning of the flow through 180° occurs. We shall consider that the flow in this section, obeying the laws of an ideal fluid, is potential. Then the problem of constructing the velocity field in the jet turning region near the cavity may be formulated as follows. We require to solve the Laplace equation

$$\Delta\Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{1}{y}\frac{\partial \Phi}{\partial y} \tag{4}$$

with the following boundary conditions:

1. At the jet turning section MM' $\partial\Phi/\partial x = U_m(y)$.
2. On the side walls of the cavity $\partial\Phi/\partial y = 0$.
3. At the surface of the bottom of the cavity $\partial\Phi/\partial x = 0$.

For convenience of further examination of the problem, we shall go over to dimensionless quantities, putting

$$\bar{U}_y = U_y/U_0; \quad \bar{U}_x = U_x/U_0; \quad \bar{x} = x/R; \quad \bar{y} = y/R.$$

Then (4) takes the form

$$\frac{\partial^2 \bar{\Phi}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{\Phi}}{\partial \bar{y}^2} + \frac{1}{\bar{y}}\frac{\partial \bar{\Phi}}{\partial \bar{y}} = 0. \tag{5}$$

The boundary conditions in dimensionless form may be written

$$\left(\frac{\partial \bar{\Phi}}{\partial \bar{x}}\right)_{\bar{x}=\bar{x}_M} = \bar{U}_M(\bar{y}) = \begin{cases} \text{from (1)}, & 0 \leqslant \bar{y} \leqslant \bar{b}_M; \\ \bar{U}_n, & \bar{b}_M \leqslant \bar{y} \leqslant 1 \end{cases}$$

$$\left(\frac{\partial \bar{\Phi}}{\partial \bar{x}}\right)_{\bar{x}=0} = 0,$$

$$\left(\frac{\partial \bar{\Phi}}{\partial \bar{y}}\right)_{\bar{y}=1} = 0. \tag{6}$$

We seek a solution of the original equation (5) with boundary conditions (6) in the form of an expansion in series with respect to Bessel functions

$$\bar{\Phi} = 2\sum_{k=0}^{\infty} \frac{\Psi_k \operatorname{ch}(\xi_k \bar{x}) J_0(\xi_k \bar{y})}{\xi_k \operatorname{sh}(\xi_k \bar{x}_M) J_0^2(\xi_k)}, \tag{7}$$

where

$$\Psi_k = \int_0^1 \bar{y}\bar{U}_M(\bar{y}) J_0(\xi_k \bar{y})\, d\bar{y}; \tag{8}$$

ξ_k are the roots of the equation $J_1(\xi) = 0$ ($\xi_0 = 0$; $\xi_1 = 3.83171$; $\xi_2 = 7.01559$; $\xi_3 = 10.17$ and so on).

Correspondingly, $\partial\Phi/\partial x$ and $\partial\Phi/\partial y$ may be written in the form

$$\frac{\partial \bar{\Phi}}{\partial \bar{x}} = \bar{U}_x = 2\sum_{k=0}^{\infty} \frac{\Psi_k \operatorname{sh}(\xi_k \bar{x}) J_0(\xi_k \bar{y})}{\operatorname{sh}(\xi_k \bar{x}_M) J_0^2(\xi_k)},$$

$$\frac{\partial \bar{\Phi}}{\partial \bar{y}} = \bar{U}_y = -2\sum_{k=0}^{\infty} \frac{\Psi_k \operatorname{ch}(\xi_k \bar{x}) J_1(\xi_k \bar{y})}{\operatorname{sh}(\xi_k \bar{x}_M) J_0^2(\xi_k)}. \tag{9}$$

Taking account of (1) and (2), the expression for Ψ_k may be rewritten as follows:

$$\Psi_k = \bar{U}_n \varepsilon_1 + (\bar{U}_m - \bar{U}_n)\left[\varepsilon_2 - \frac{19.5}{(\bar{x}_0 - \bar{x}_M)^{1.5}}\varepsilon_3 + \frac{95}{(\bar{x}_0 - \bar{x}_M)^{1.5}}\varepsilon_4\right], \tag{10}$$

where

$$\varepsilon_1 = \int_0^1 \bar{y} J_0(\xi_k \bar{y})\, d\bar{y}, \quad \varepsilon_2 = \int_0^{\bar{b}_M} \bar{y} J_0(\xi_k \bar{y})\, d\bar{y},$$

$$\varepsilon_3 = \int_0^{\bar{b}_M} \bar{y}^{-2.5} J_0(\xi_k \bar{y})\, d\bar{y}, \quad \varepsilon_4 = \int_0^{\bar{b}_M} \bar{y}^4 J_0(\xi_k \bar{y})\, d\bar{y},$$

$$\bar{b}_M = \frac{b_M}{R} = 0.22(\bar{x}_0 - \bar{x}_M).$$

In the event that expressions from (3) are used for $\bar{U}_m$, $\bar{U}_n$, and $\bar{b}_M$, calculation of $\bar{U}_x$ and $\bar{U}_y$ in the jet turning region presents no appreciable difficulty. It should be noted, however, that the use of expression (1) for the velocity profile in the jet turning section MM' leads in this case to a very laborious calculation. This latter disadvantage may be avoided to a considerable extent by using, as an expression for the velocity profile in section MM', an expansion of the type

$$\frac{\bar{U}_x - \bar{U}_n}{\bar{U}_m - \bar{U}_n} = a_0 + a_1 \bar{y} + a_2 \bar{y}^2 \tag{11}$$

instead of (1).

The coefficients of the expansion are found from the following conditions:

$$\bar{y} = 0, \quad \bar{U}_x = \bar{U}_m, \quad \frac{\partial \bar{U}_x}{\partial \bar{y}} = 0,$$

$$\bar{y} = \bar{b}_M, \quad \bar{U}_x = \bar{U}_n.$$

In this case the expression for Ψ_k is written as

$$\Psi_0 = 0.5\,\bar{U}_n + 0.25\,\bar{b}_M^2 (\bar{U}_m - \bar{U}_n),$$

$$\Psi_k = \frac{2(\bar{U}_m - \bar{U}_n)}{\xi_k^2} J_2(\xi_k \bar{b}_M), \quad k = 1, 2, 3, \ldots,$$

and, correspondingly, $\bar{U}_x$ and $\bar{U}_y$ take the form

$$\bar{U}_x = [\bar{U}_n + 0.5\bar{b}_M^2 (\bar{U}_m - \bar{U}_n)] \frac{\bar{x}}{\bar{x}_M} + 4(\bar{U}_m - \bar{U}_n) \times \sum_{k=1}^{\infty} \frac{J_2(\xi_k \bar{b}_M)\, \mathrm{sh}(\xi_k \bar{x}) J_0(\xi_k \bar{y})}{\xi_k^2 J_0^2(\xi_k)\, \mathrm{sh}(\xi_k \bar{x}_M)},$$

$$\bar{U}_y = -0.5 [\bar{U}_n + 0.5\bar{b}_M^2 (\bar{U}_m - \bar{U}_n)] \frac{\bar{y}}{\bar{x}_M} - 4(\bar{U}_m - \bar{U}_n) \times \sum_{k=1}^{\infty} \frac{J_2(\xi_k \bar{b}_M)\, \mathrm{ch}(\xi_k \bar{x}) J_1(\xi_k \bar{y})}{\xi_k^2 J_0^2(\xi_k)\, \mathrm{sh}(\xi_k \bar{x}_M)}. \tag{12}$$

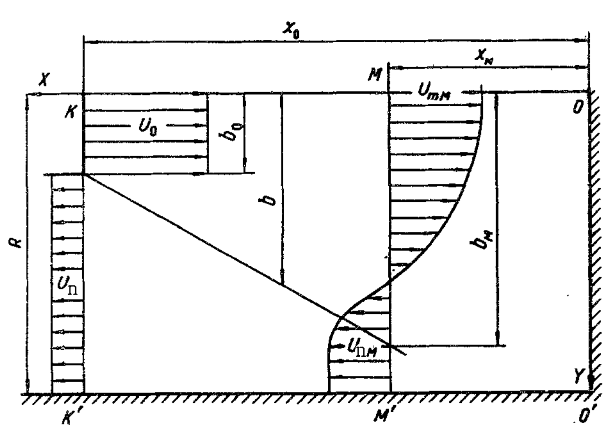

Fig. 1. Schematic of the flow in the axisymmetric closed cavity.

If the profile (11) is considered to extend over the whole jet turning section M'M, i. e., if we take $\bar{b}_M \simeq 1$, the expression for Ψ_k takes the form

$$\Psi_0 = 0.25(\bar{U}_n + \bar{U}_m),$$

$$\Psi_k = -\frac{2(\bar{U}_m - \bar{U}_n)}{\xi_k^2} J_0(\xi_k), \quad k = 1, 2, 3, \ldots.$$

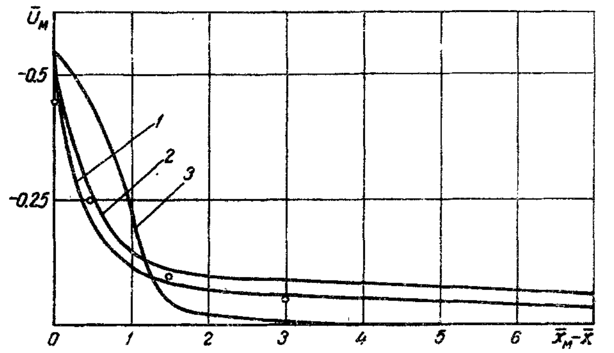

Fig. 2. Variation of dimensionless value of stream velocity along the axis of the axisymmetric cavity at the section OM: 1—according to (12); 2—(13); 3—calculated in [1]; the points—experiment in [1].

Then the expressions for $\bar{U}_x$ and $\bar{U}_y$ are written as

$$\bar{U}_x = 0.5(\bar{U}_n + \bar{U}_m) \frac{\bar{x}}{\bar{x}_M} - 4(\bar{U}_m - \bar{U}_n) \times \sum_{k=1}^{\infty} \frac{\mathrm{sh}(\xi_k \bar{x}) J_0(\xi_k \bar{y})}{\xi_k^2 J_0(\xi_k)\, \mathrm{sh}(\xi_k \bar{x}_M)}, \quad \bar{U}_y = -0.25(\bar{U}_n + \bar{U}_m) \frac{\bar{y}}{\bar{x}_M} + 4(\bar{U}_m - \bar{U}_n) \sum_{k=1}^{\infty} \frac{\mathrm{ch}(\xi_k \bar{x}) J_1(\xi_k \bar{y})}{\xi_k^2 J_0(\xi_k)\, \mathrm{sh}(\xi_k \bar{x}_M)}. \tag{13}$$

Curves 1 and 2 of Fig. 2 were calculated from (12) and (13), taking account of the relations for $\bar{U}_n$, $\bar{U}_m$, and $\bar{b}_M$ from (3) for a nozzle diameter $\bar{b}_0 = 0.186$ and $\bar{x}_M = 15.15$. The theoretical curve of [1] was obtained for the case of flow in a plane cavity.

NOTATION

U_x, U_y are the longitudinal and transverse velocity components; U_0, U_m, U_n is the longitudinal velocity component, respectively, at the rim of the nozzle, on the axis of the jet, and on the side surface of the cavity; U_{mM}, U_{nM} is the longitudinal velocity component at the jet turning section M'M on the jet axis, and on the side surface of the cavity; x, y are the longitudinal and transverse coordinates; x_0, x_M are the coordinates of the nozzle rim and of the jet turning line M'M; b_0 is the nozzle diameter; b_M is the thickness of the mixing zone at the jet turning section; R is the radius of the cavity; Φ is the velocity potential.

REFERENCES

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